SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 2, SS 2018
Sheet 5

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1. (a) Which of the subsets

$$\begin{aligned} U_1 &= \{p \in \mathcal{P}(\mathbb{R}) : p(0) = 0\}, \\ U_2 &= \{p \in \mathcal{P}(\mathbb{R}) : p(0) = 1\}, \\ U_3 &= \{p \in \mathcal{P}(\mathbb{R}) : p(1) = 0\}, \\ U_4 &= \left\{p \in \mathcal{P}(\mathbb{R}) : \int_0^1 p(x)\,\mathrm{d}x = 0\right\}, \\ U_5 &= \{p \in \mathcal{P}(\mathbb{R}) : p'(0) + p''(0) = 0\}, \\ U_6 &= \{p \in \mathcal{P}(\mathbb{R}) : p'(0)p''(0) = 0\} \end{aligned}$$

of $\mathcal{P}(\mathbb{R})$ are subspaces of $\mathcal{P}(\mathbb{R})$?

(b) Which of the subsets

$$S_1 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a = b\right\}, \quad S_2 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b = 1\right\}, \quad S_3 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a^2 = b^2\right\}$$

of $\mathbb{R}^{2\times 2}$ are subspaces of $\mathbb{R}^{2\times 2}$?

2. (a) Show that $\{x^3 - x^2, x^3 - x\}$ is a basis for the subspace

$$W = \{p \in \mathcal{P}_3(\mathbb{R}) : p(0) = p(1) = 0\}$$

of $\mathcal{P}_3(\mathbb{R})$. Extend this basis to a basis for $\mathcal{P}_3(\mathbb{R})$ and hence find a complement of W in $\mathcal{P}_3(\mathbb{R})$.

(b) Let S_3 be the set of all real, symmetric 3×3 matrices. Find a basis for the subspace S_3 of $\mathbb{R}^{3\times 3}$ and hence determine $\dim S_3$. Extend this basis to a basis for $\mathbb{R}^{3\times 3}$ and hence find a complement of S_3 in $\mathbb{R}^{3\times 3}$.

[Note: An $n \times n$ matrix $A = (a_{ij})_{i,j=1,\ldots,n}$ is called *symmetric* if $a_{ij} = a_{ji}$ for all $i, j = 1, \ldots n$.]

3. (a) Let U_1 and U_2 be subspaces of a vector space V. Show that $U_1 \cup U_2$ is a subspace of V if and only if $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.

(b) Let U_1 and U_2 be subspaces of a vector space V. Show that $U_1 + U_2$ is a subspace of V.