SAARLAND UNIVERSITY
Department of Mathematics
Prof. Dr. Mark Groves
MSc Jens Horn

Mathematics for Computer Scientists 2, SS 2018
Sheet 8

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1. Determine for which values of $\lambda \in \mathbb{R}$ the real matrix

$$A_\lambda = \begin{pmatrix} 1 & \lambda & 0 & 0 \\ \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \end{pmatrix}$$

is invertible, and compute the inverse matrix A_λ^{-1} for these values of λ.

2. Let

$$B = \begin{pmatrix} 2 & 1 & 1 & 1 & 2 \\ 3 & 2 & 1 & 1 & 2 \\ 4 & 2 & 2 & 3 & 5 \\ 2 & 1 & 1 & 2 & 3 \end{pmatrix} \in \mathbb{R}^{4\times 5}$$

and $r = \operatorname{Rang} B$. Find matrices $T \in \mathrm{GL}(4, \mathbb{R})$ and $S \in \mathrm{GL}(5, \mathbb{R})$ such that

$$T^{-1}BS = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}. \qquad (\star)$$

[Hint: First convert B into echelon form using elementary row operations, then convert the resulting matrix into the form $(\star)$ using elemenary column operations. The matrix S is obtained by applying the column operations to I_5 in the same order, while the matrix T is obtained by applying the row operations to I_4 in reverse order.]

3. Let p be a prime number. Determine whether the matrix

$$C = \begin{pmatrix} 13 & 7 & 6 \\ -7 & 1 & 1 \\ 3 & 8 & 7 \end{pmatrix} \in \mathbb{Z}_p^{3\times 3}$$

is invertible in the cases $p = 2$, $p = 3$ and $p = 5$, and compute C^{-1} if it exists.

4. Construct a 4×4 real D such that

$$\ker D = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle, \qquad \operatorname{Im} D = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle.$$

[Hint: $D = (D\mathbf{e}_1 | D\mathbf{e}_2 | D\mathbf{e}_3 | D\mathbf{e}_4)$ and $\operatorname{Im} D = \langle D\mathbf{e}_1, D\mathbf{e}_2, D\mathbf{e}_3, D\mathbf{e}_4 \rangle$.]