SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 2, SS 2018
Sheet 2

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1. Compute the following indefinite integrals.

(i) $\displaystyle\int \frac{(1+x)^2}{\sqrt{x}}\,\mathrm{d}x$

(ii) $\displaystyle\int x\sqrt{x+9}\,\mathrm{d}x$

(iii) $\displaystyle\int \frac{x}{(x-1)(x-2)}\,\mathrm{d}x$

[Hint: use partial fractions.]

(iv) $\displaystyle\int \frac{1}{x(x-1)(x-2)}\,\mathrm{d}x$

[Hint: use partial fractions.]

(v) $\displaystyle\int \frac{8-x}{(x-2)^2(x+1)}\,\mathrm{d}x$

[Hint: use partial fractions.]

(vi) $\displaystyle\int \frac{1}{x^2+8x+25}\,\mathrm{d}x$

[Hint: complete the square and use the substitution $x = -4 + 3\tan\theta$.]

(vii) $\displaystyle\int \sqrt{1+16x^2}\,\mathrm{d}x$

[Hint: use the substitution $x = \frac{1}{4}\sinh\theta$.]

(viii) $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x$

[Hint: use the substitution $x = \sin\theta$.]

2. Let $I_n = \displaystyle\int \cos^n x\,\mathrm{d}x$ for $n = 2, 3, \ldots$.

(i) Show that

$$nI_n = \cos^{n-1} x \sin x + (n-1)I_{n-2} + c, \qquad n = 0, 1, 2, \ldots.$$

(ii) Compute $\displaystyle\int_0^{\pi/2} \cos^7 x\,\mathrm{d}x$.

3. Compute the following indefinite integrals.

(i) $\displaystyle\int \sin^2 2x \cos^3 2x \, \mathrm{d}x$

(ii) $\displaystyle\int \tan^2 x \, \mathrm{d}x$

(iii) $\displaystyle\int \frac{\sin x}{(1+\cos x)^2} \, \mathrm{d}x$

(iv) $\displaystyle\int \frac{\mathrm{e}^x}{1+\mathrm{e}^x} \, \mathrm{d}x$

(v) $\displaystyle\int \frac{\sin x + 2\cos x}{\cos x + 2\sin x} \, \mathrm{d}x$

[Hint: note that

$$\sin x + 2\cos x = \frac{3}{5}(-\sin x + 2\cos x) + \frac{4}{5}(\cos x + 2\sin x)$$

for all $x \in \mathbb{R}$.]

(vi) $\displaystyle\int \frac{1}{3+5\cos x} \, \mathrm{d}x$

[Hint: note that

$$\cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$$

for all $x \in \mathbb{R}$ and use the substitution $t = \tan \frac{x}{2}$.]

(vii) $\displaystyle\int \frac{1}{25-24\sin^2 x} \, \mathrm{d}x$

[Hint: note that

$$\sin^2 x = \frac{\tan^2 x}{1+\tan^2 x}$$

for all $x \in \mathbb{R}$ and use the substitution $t = \tan x$.]