SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2018/19
Sheet 11

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1. (a) How many n-digit natural numbers without the digit 9 are there?

(b) Prove that the sum of the reciprocal values of the n-digit natural numbers without the digit 9 is less than or equal to $8(\frac{9}{10})^{n-1}$.

(c) Prove that the series obtained from the harmonic series by removing those summands with the digit 9 in their denominator is convergent.

2. Give rigorous formulations of the following statements.

(i) $f(x) \to \infty$ for $x \to \infty$

(ii) $f(x) \to -\infty$ for $x \to \infty$

(iii) $f(x) \to \infty$ for $x \to -\infty$

(iv) $f(x) \to -\infty$ for $x \to -\infty$

(v) $f(x) \to \infty$ for $x \to a$

(vi) $f(x) \to -\infty$ for $x \to a$

3. (a) Let $\lim_{x\to a} f(x) = \ell$. Prove that $\{f_n\}$ converges to ℓ for every sequence $\{x_n\}$ with $x_n \neq a$ which converges to a.

(b) Prove the converse to (a) by contradiction: suppose that $\{f(x_n)\}$ converges to ℓ for every sequence $\{x_n\}$ with $x_n \neq a$ which converges to a, set $\delta = 1/n$, $n = 1, 2, 3, \ldots$ in the rigorous formulation of the statement '$\lim_{x\to a} f(x) \neq \ell$' and find a sequence $\{x_n\}$ with $x_n \neq a$ and $x_n \to a$ but $f(x_n) \not\to \ell$ as $n \to \infty$.

(c) Suppose that the functions $f, g : \mathbb{R} \to \mathbb{R}$ are continuous. Prove that $f \circ g : \mathbb{R} \to \mathbb{R}$ is also continuous.

(d) Suppose that the functions $f, g : \mathbb{R} \to \mathbb{R}$ are continuous and $f(q) = g(q)$ for all rational numbers q. Prove that $f(x) = g(x)$ for all real numbers x.

4. This exercise is to be solved using the intermediate-value theorem.

(i) Let $\alpha < \beta$. Show that the equation

$$\frac{x^2+1}{x-\alpha} + \frac{x^6+1}{x-\beta} = 0$$

has at least one solution $x_0 \in (\alpha, \beta)$.

(ii) Show that the equation

$$2^x = 4x$$

has at least one solution other than $x = 4$.