SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2018/19
Sheet 3

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1. The following tables show the results of the arithmetical operations in $\mathbb{Z}_3$ (where $\oplus$ and $\odot$ denote addition and multiplication modulo 3).

$\oplus$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[1]$	$[2]$
$[1]$	$[1]$	$[2]$	$[0]$
$[2]$	$[2]$	$[0]$	$[1]$

$\odot$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[0]$	$[0]$
$[1]$	$[0]$	$[1]$	$[2]$
$[2]$	$[0]$	$[2]$	$[1]$

(a) Compute the corresponding tables for $\mathbb{Z}_5$ and $\mathbb{Z}_7$.

(b) Compute the corresponding tables for $\mathbb{Z}_4$ and show that $(\mathbb{Z}_4, \oplus, \odot)$ is not a field.

2. Show that $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a subfield of $(\mathbb{R}, +, .)$.

3. Show that $\mathbb{C}$ is not an ordered field with respect to the usual addition and multiplication. [Hint: Show that the assumptions $0 < \mathrm{i}$ and $\mathrm{i} < 0$ both lead to contradictions.]

4. Define the binary operations 'subtraction' and 'division' on a field $(K, +, .)$. Let a, b, c, d be Elements of K with b, $d \neq 0$. Show that

$$\frac{a}{b} - \frac{c}{d} = \frac{a.d - b.c}{b.d}, \qquad \frac{a}{b} \bigg/ \frac{d}{c} = \frac{a.c}{b.d},$$

using *only the axioms of arithmetic and your definitions.*

5. Let X be a nonempty set and $\cdot$ an associative binary operation on X with the following properties.

(i) The element $e \in X$ satisfies $e \cdot x = x$ for all $x \in X$.

(ii) For each $x \in X$ there exists an element x^{-1} with $x^{-1} \cdot x = e$.

Show that $x \cdot e = x$ and $x \cdot x^{-1} = e$ for all $x \in X$.