SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2017/18
Sheet 7

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1. Prove using the prime decomposition theorem that $\sqrt{n}$ is irrational for each $n \in \mathbb{N}$ with $n \neq m^2$ for some $m \in \mathbb{N}$.

2. a) Show that the sum of a irrational number and a rational number is irrational.

b) Show that the product of an irrational number and a non-zero rational number is irrational.

c) Give a counterexample to the assertion that the sum and product of two irrational numbers is rational.

d) Give a counterexample to the assertion that the sum and product of two irrational numbers is irrational.

3. Show that $\mathbb{C}$ is not an ordered field with respect to the usual addition and multiplication. [Hint: Show that the assumptions $0 < \mathrm{i}$ and $\mathrm{i} < 0$ both lead to contradictions.]

3. Let

$$\begin{aligned} A &= \{z \in \mathbb{C} : |z - 2 - 3\mathrm{i}| < |z + 4 - 5\mathrm{i}|\}, \\ B &= \{z \in \mathbb{C} : 0 \leq \arg(z + 3 - 4\mathrm{i}) < \pi/4\}. \end{aligned}$$

Sketch the set $A \cap B$.

4. Find all complex solutions to the following equations.

(a) $3z^2 + z = 1$

(b) $z^2 - (3 + \mathrm{i})z + 4 + 3\mathrm{i} = 0$

(c) $\sinh z = \mathrm{i}$

(d) $z^2 + 2\bar{z}^2 + z - \bar{z} + 9 = 0$

(e) $z^4 - 4z^2 + 16 = 0$

(f) $z^4 + 1 = 0$

(g) $(z^2 - 1)^3 = 8z^3$

(h) $z^6 - 3\mathrm{i}z^3 - 2 = 0$

(i) $z^3 + 2z^2 + 2z = 0$

(j) $z^3 - (3 + \mathrm{i})z^2 + (2 + 3\mathrm{i})z - 2\mathrm{i} = 0$

(k) $\mathrm{e}^z = \mathrm{e}^{\mathrm{i}z}$

(l) $\mathrm{e}^{2z} + \mathrm{i}\mathrm{e}^z + 1 = 0$

5. Compute $(4\sqrt{3} - 4\mathrm{i})^{88}$. [Hint: use de Moivre's theorem.]